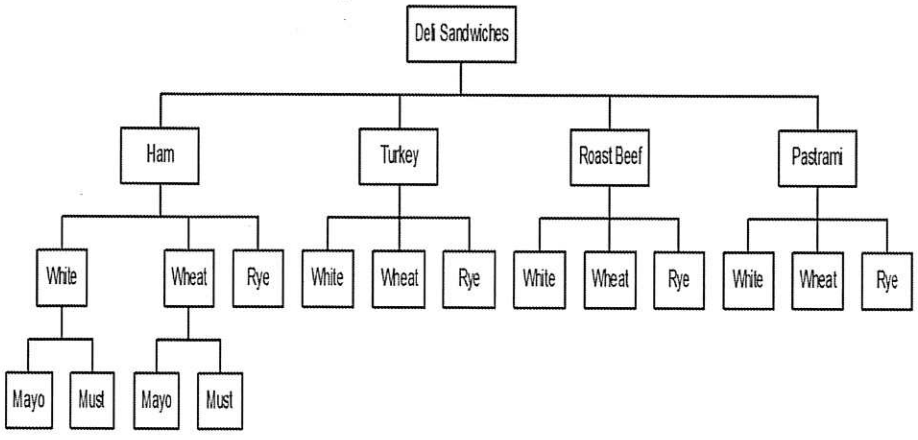
ALGEBRA II
10.1 FUNDAMENTAL COUNTING PRINCIPAL AND PERMUTATIONS

Obj: to find the number of ways an event can occur using the fundamental counting principal and permutations.

How many different sandwiches can a deli offer if a customer may choose from ham, turkey, roast beef or pastrami; white, wheat or rye bread; and mayo. or mustard?



Fundamental Counting Principal:

If one event can occur in m ways and another event can occur in n ways, then the number of ways both events can occur is mn ways.

If you flip a coin and roll a die, how many outcomes are there?

$2 \times 6 = 12$

If you roll two different dice, how many outcomes are there?

$6 \times 6 = 36$

At a restaurant you have a choice of 8 different entrees, 2 different salads, 12 different drinks, and 6 different deserts. How many dinners of one of each of the above choices would there be?

$8 \times 2 \times 12 \times 6 = 1,152$

A standard configuration for a New York license plate is 3 digits followed by 3 letters.

How many different plates are there if the digits and numbers can be repeated?

$10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 = 17576000$

How many different plates are there if the digits and numbers can not be repeated?

$10 \cdot 9 \cdot 8 \cdot 26 \cdot 25 \cdot 24 = 11232000$

How many different seven digit phone numbers are there if the first digit cannot be a 0 or 1?

$8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 =$

$8{,}000{,}000$

Facorial:

$6! = 720$ $(6 \times 5 \times 4 \times 3 \times 2 \times 1)$

$4! = 24$ $(4 \times 3 \times 2 \times 1)$

$5! = 120$

$10! = 3628800$

$0! = 1$

Why is zero factorial equal to one? 0! = 1

Proof:

Start with	$n! = n * (n-1)!$
Switch sides of the equation	$n * (n-1)! = n!$
Divide both sides by n	$\frac{n}{n} * (n-1)! = \frac{n!}{n}$
	$(n-1)! = \frac{n!}{n}$
Replace n with 1.	$(1-1)! = \frac{1!}{1}$
Simplify	$0! = 1$

Permutation: An ordering of objects.

How many permutations of the letters A, B & C are there?

ABC BAC CAB
ACB BCA CBA 6

Twelve people are competing in a race. How many different ways could they finish? How many different ways can 3 of the people finish in first, second and third places?

$12! = 479,001,600$

$12 \cdot 11 \cdot 10 = 1320$

Permutations of n objects taken r at a time:

$${}_nP_r = \frac{n!}{(n-r)!}$$

${}_6P_2 =$ 30

${}_6P_4 =$ 360 ${}_6P_6 =$ 720 ${}_6P_1 =$ 6

How many different ways can 3 of 12 people finish in first, second and third places?

${}_{12}P_3$ 1320

You are considering 10 different colleges.

In how many ways (orders) could you visit 6 of them?

${}_{10}P_6 = 151200$

In how many ways (orders) could you visit 10 of them?

${}_{10}P_{10}$ or $10! = 3,628,800$

Find the number of distinguishable permutations of the letters in each word. (How many different arrangements of the letters can you make?)

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DOG	TIRE	WELSH
$3!$	$4!$	$5!$

Permutations with repetition.
When you have n objects and one is repeated q_1 times another is repeated q_2 times…

$$Permutations = \frac{n!}{q_1!q_2!q_3!\ldots}$$

How many distinguishable permutations of the following words are there?

HOPEWELL	MISSISSIPPI
$\frac{8!}{(2!\cdot 2!)}$	$\frac{11!}{(4!\cdot 4!\cdot 2!)}$